The idea of this course:

Filling in the gaps for things you (hopefully!) alreddy know

Kronecker (19th century mathematician)

on natural numbers:

"God made the natural numbers.

All else is the work of man."

Natural numbers = counting numbers

1, 2, 3, 4, 5, ... —

Notation for the set of natural

numbers: $\mathbb{N}$

The Babylonians apparently introduced zero before the Greeks ($\sim$ 400 BC) and negative integers by the Chinese ($\sim$ 200 BC)

Notation: ($\mathbb{Z}$ and $\mathbb{Q}$)

$\mathbb{Z}$ = all numbers in $\mathbb{N}$, plus their negatives, and zero.

$\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots$

$\mathbb{Q}$ = rational numbers, all ratios of numbers in $\mathbb{Z}$ (no division by zero) e.g. $1, -5 \in \mathbb{Q}$

$(\mathbb{Z} \subset \mathbb{Q})$, but we

also have $\frac{1}{2} \in \mathbb{Q}$,

but $\frac{1}{2} \notin \mathbb{Z}$.

Are there numbers

that aren't rational,

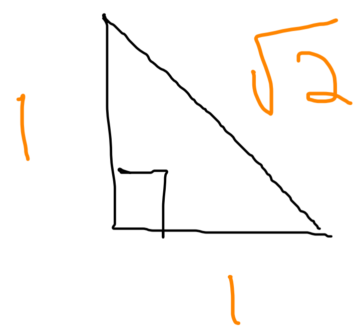
i.e., is there a number

that is not in $\mathbb{Q}$?

For a while, the Greeks

thought not.

Take a right triangle



By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$.

Pythagorean era

$\sqrt{2}$ is irrational

(proof tomorrow)

Note: there are infinitely many irrational numbers ($\sqrt{p}$ for p a prime, and there are infinitely many primes - proof later today!)

Just how "many" irrational numbers there are had to wait until Cantor – ostracized by Kronecker!

We want to understand Cantor's proof that the irrational numbers are "bigger" than the rationals.

Mathematical Proof (review)

You are given a statement such as "There are infinitely many prime numbers." This is true. What is meant to "prove" it?

Well, ——

2, 3, 5, 7, 11, 13, 17, 19, 23, ...

This argument lacks coherence and finiteness.

What kind of convincing argument could we make?

We need to reason from what we already know, and provide a complete argument.

What you can't do:

Assume anything that is a consequence of the statement you're trying to prove

ESPECIALLY the conclusion of the statement!

Methods of Proof

1) Proof by Contradiction.

Assume the negation of the statement you're trying to prove.
Reason from this statement to a fact you know to be false.

Theorem: There are infinitely many prime numbers.

Proof: Suppose, by contradiction, that there are only finitely many primes $p_1, p_2, \ldots, p_n$ for $n \in \mathbb{N}$.

Set $q = (p_1 \cdot p_2 \cdot p_3 \cdot \_\_ \cdot p_n) + 1$.

q is not divisible by any of the primes $p_1, p_2, \ldots, p_n$, so there must be another prime.

This is a contradiction since we had assumed there are only n primes and now we have n+1.

Therefore, there must be infinitely many primes. $\square$